Chaplygin Gas with Power-Law Dark Energy

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ABSTRACT

Our universe is under the accelerating expansion phase. Many models have been proposed to explain this behavior. Among these models, the power-law and Chaplygin gas are two of the most interesting models. We studied the Chaplygin gas in the scenario of canonical power-law (CGP model) and phantom power-law (CGPP model). In these two models, the deceleration parameter (q_0), the power-law exponents and the equation of state parameter (w_0) at present are calculated. In this calculation, we used two observational data coming from WMAP9 (WMAP9+eCMB+BAO+H_0) and PLANCK satellite reported in 2018 (*TT,TE,EE*+ lowE+Lensings+BAO). The results shown that the CGP model does not correspond to accelerating expansion conditions. In the CGPP model, the results shown that the CGPP model is correspond to accelerating expansion conditions. Finally, the values of the equation of state parameter coming from both CGP and CGPP models are the same and match all observational data under certain conditions.

Keywords: *Dark Energy, Accelerating Universe, Chaplygin Gas, Power-Law*

INTRODUCTION

Our present universe is under the accelerating expansion phase as observed in the Type Ia supernovae (SNIa) (Amanullah et al., 2010; Astier et al., 2006; Goldhaber et al., 2001; Perlmutter et al., 1998 and 1999; Riess et al., 1998, 1999, 2004 and 2007; Tonry et al., 2003), the study of large-scale structure (LSS) (Scranton et al., 2003; Tegmark et al., 2004) the cosmic microwave background (CMB) (Larson et al., 2011; Komatsu et al., 2011; Hinshaw et al., 2013; Bennett et al., 2013; Hu et al., 2014; Masi et al., 2002) and the X-ray luminosity from galaxy cluster (Hu et al., 2014; Allen et al., 2004; Rapetti et al. 2005). Cosmologists and physicists believe that it is correspond to the unknown source of energy called dark energy (Copeland et al., 2006; Padmanabhan, 2005 & 2006; Amendola & Tsujikawa, 2010). Many models have been proposed to explain this behavior of the universe, such as quintessence (Dutta et al., 2009; Liddle & Scherrer, 1999; Ratra & Peebles, 1988; Wetterich, 1988), tachyon (Garousi, 2000; Rangdee & Gumjudpai, 2014; Sen, 2002a & 2002b), k-essence (Armendariz-Picon & Steinhardt, 2000 & 2001), Chaplygin gas (CG) (Colistete Jr. et al., 2002; Dev et al., 2003; Gorini et al., 2003 & 2006; Saha et al., 2017) models. In the universe history, there were epoch that radiation or matter is

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dominant component in the universe for which the scale factor evolves as power-law. The power-law model (Dev et al., 2008; Gumjudpai, 2013; Kumar, 2012; Jain et al., 2003; Zhu et al., 2008), one of the most interesting models which the scale factor a scaled as a power of time t, can be used to describe the universe in the radiation dominated epoch ($a \propto t^{1/2}$) and in the matter dominated epoch ($a \propto t^{2/3}$). This model also avoids the horizon and the flatness problems. Therefore, a present universe with mixed combination of many different ingredients and dominated by dark energy can be possible to describe by power-law cosmology model.

In the first part of this work, we considered our universe having a standard flat FLRW space filled with dust-matter and dark energy in form of Chaplygin gas. In the second part, we studied the CG model and the canonical and phantom power-law cosmologies. In the canonical (phantom) power-law (Rangdee & Gumjudpai, 2014; Dev et al., 2008; Gumjudpai, 2013; Kumar, 2012; Jain et al., 2003; Zhu et al., 2008), the scale factor is scale as $a(t) \propto t^{\alpha}$ ($a(t) \propto (t_s - t)^{\beta}$) corresponding to acceleration if $\alpha > 1$ ($\beta < 0$) with $0 < \alpha < 1$ ($-1 < \beta < 0$). The third part, we examined the scenario of Chaplygin gas mix with canonical power-law (CGP) and with phantom power-law (CGPP). We aimed to test whether these two models can be used to describe the present situation and to predict the future of our universe or not. The results of the calculation of the cosmological parameters are presented in the fourth part including discussion. In these calculations, we used the observational data from WMAP9 (WMAP9+ eCMB+BAO+H_0) (Hinshaw et al., 2013) and from PLANCK satellite reported in 2018 (*TT*,*TE*,*EE*+lowE+Lensings+BAO) (Aghanim et al., 2020) or PLANCK2018. Those parameters are the deceleration parameter (q_0), the power exponents (α and β), and the equation of state parameter (w_0). Therefore, we have analyzed and compared the results with those observational data in this section. Finally, in the last part we conclude that whether the CGP and the CGPP models can be used to describe the current situation of our universe and whether they can be used to predict the future fate of our universe or not.

CHAPLYGIN GAS AND CANONICAL (PHANTOM) POWER-LAW COSMOLOGY

The Chaplygin gas (Colistete Jr. et al., 2002; Dev et al., 2003; Gorini et al., 2003 & 2006; Saha et al., 2017) model combines both dark energy and cold dark matter together. CG model behaves like cold dark matter at small scale and has the negative pressure behavior at large scale. This CG model gives the results that correspond to the accelerating expansion universe at late time. Its pressure can be written in the simple form as

$$p_{CG} = -\frac{A}{\rho_{CG}}, \tag{1}$$

where A is a positive constant. This model corresponds to the tachyon model when the constant A act as a constant potential square; that is, $p = -V^2(\phi)/\rho$. In other words, CG model is a special case of tachyon model with constant potential. By using the continuity equation, we can write the energy density of the CG model as follow:

$$\rho_{CG} = \sqrt{A + \frac{B}{a^6}}, \tag{2}$$

where B is an integral constant. Then Equation (1) can be rewritten as

$$p_{CG} = -\frac{A}{\sqrt{A + \frac{B}{a^6}}}, \tag{3}$$

and the equation of state parameter w_{CG} is

$$w_{CG} \equiv \frac{p_{CG}}{\rho_{CG}} = -\frac{A}{A + \frac{B}{a^6}}. \tag{4}$$

To evaluate this equation, we consider two cases. First, scale factor $a \ll (B/A)^{1/6}$ corresponding to the early universe with small scale, the CG model behaves like the cold dark matter and then its pressure and energy density reduce to $p_{CG} \approx -(Aa^3)/\sqrt{B}$ and $\rho_{CG} \approx \sqrt{B}/a^3$, respectively. Then the equation of state parameter is $w_{CG} \approx -A/(Ba^6)$. Another case, scale factor $a \gg (B/A)^{1/6}$ corresponding to late time with large scale universe. This case the CG model has a negative pressure $p_{CG} \approx -\sqrt{A} = -\rho_{CG}$, and hence the equation of state can be reduced to $w_{CG} \approx -1$ corresponding to the accelerating expansion behavior.

Besides the equation of state parameter Equation (4), we can weigh the dust-matter content by the effective equation of state parameter $w_{eff,CG} \equiv \rho_{CG} w_{CG}/(\rho_{CG} + \rho_m)$. With all information above we can express $w_{eff,CG}$ as

$$w_{eff,CG} = -\frac{A}{A + \frac{B}{a^6} + \rho_m \sqrt{A + \frac{B}{a^6}}} = \frac{w_{CG}}{1 + \rho_m \sqrt{A + \frac{B}{a^6}}}. \tag{5}$$

Power-law cosmology comes from the solution of the Friedmann equation with dark energy in flat universe, $H^2 = 8\pi G\rho/3$, with a constant equation of state w. For $-1/3 > w > -1$, the solution gives power-law form,

$$a(t) = a_0 \left(\frac{t}{t_0}\right)^{\alpha}, \tag{6}$$

where $a_0 = a(t = t_0)$ is a scale factor at present, α is a constant with $0 < \alpha < \infty$. In power law cosmology, the speed and acceleration are $\dot{a} = \alpha a/t$ and $\ddot{a} = \alpha(\alpha - 1)a/t^2$, respectively. The Hubble parameter is $H(t) = \dot{a}/a = \alpha/t$ and we can calculate the power exponent (α) at present from $\alpha = H_0 t_0$. Here H_0 is the Hubble constant, the Hubble parameter at present. The deceleration parameter is $q \equiv -a\ddot{a}/\dot{a}^2 = (1/\alpha) - 1$. Since $\alpha > 0$, we then have $q \geq -1$ and $H_0 \geq 0$. Many observations (Dev et al., 2008; Gumjudpai, 2013; Kumar, 2012; Jain et al., 2003; Zhu et al., 2008) show that the power exponent must greater than 1 ($\alpha > 1$) to correspond with the present accelerating expansion universe.

In the case of phantom power-law cosmology, it is a case with constant equation of state parameter; that is, $w < -1$ and the phantom power-law form,

$$a(t) = a_0 \left(\frac{t_s - t}{t_s - t_0}\right)^{\beta}, \tag{7}$$

Table 1: Derived parameters from WMAP9 (WMAP9+eCMB+BAO+H_0) (Hinshaw et al., 2013) and from PLANCK satellite reported in 2018 (TT,TE,EE+lowE+Lensings +BAO) (Aghanim et al., 2020). Here we can calculate critical density: $\rho_{c,0} = 3H_0^2/8\pi G$ and dust-matter energy density: $\rho_{m,0} = \rho_{c,0}\Omega_{m,0}$. The space is flat and we set $a_0 = 1$.

Parameter	**WMAP9** (Hinshaw et al., 2013)	**PLANCK2018** (Aghanim et al., 2020)
t_0	$(4.346(4) \pm 0.018(6)) \times 10^{17}$ sec	$(4.350(7) \pm 0.006(3)) \times 10^{17}$ sec
	13.772 ± 0.059 Gyr	13.796 ± 0.020 Gyr
H_0	$(2.245(9) \pm 0.025(9)) \times 10^{-18}$ sec^{-1}	$(2.192(5) \pm 0.013(6)) \times 10^{-18}$ sec^{-1}
	69.32 ± 0.80 km/sec/Mpc	67.66 ± 0.42 km/sec/Mpc
$\Omega_{m,0}$	$0.2865^{+0.0096}_{-0.0095}$	0.3103 ± 0.0057
$\rho_{c,0}$	$(9.019(6) \pm 0.208(8)) \times 10^{-27}$ kg/m^3	$(8.597(2) \pm 0.106(6)) \times 10^{-27}$ kg/m^3
$\rho_{m,0}$	$(2.584(1)^{+0.146(4)}_{-0.145(5)}) \times 10^{-27}$ kg/m^3	$(2.667(7) \pm 0.082(1)) \times 10^{-27}$ kg/m^3
$w_{DE,0}$	$-1.073^{+0.090}_{-0.089}$	$-1.03^{+0.10}_{-0.11}$

where $t_s \equiv t_0 + |\beta|/H_0$ is a future big-rip time (Gumjudpai, 2020; Caldwell, 2002; Caldwell et al., 2003) and β is a constant power exponent with $\beta < 0$ corresponding to the present accelerating expansion universe. In this scenario, speed and acceleration are $\dot{a} = -\beta a/(t_s - t)$ and $\ddot{a} = \beta(\beta - 1)a/(t_s - t)^2$. Hence the Hubble parameter

$$H = \frac{\dot{a}}{a} = -\frac{\beta}{t_s - t}. \tag{8}$$

At present, the present power exponent is $\beta = H_0(t_0 - t_s)$ and the deceleration parameter is $q \equiv -a\ddot{a}/\dot{a}^2 = (1/\beta) - 1$. The big-rip time can be calculated from [30]

$$t_s \approx t_0 - \frac{2}{3(1 + w_{DE})}\frac{1}{H_0\sqrt{1 - \Omega_{m,0}}}, \tag{9}$$

where w_{DE} is a present equation of state parameter of dark energy and must be less than -1, $\Omega_{m,0}$ is a dimensionless density parameter of dust-matter content of a present universe. The derived parameters from WMAP9 (WMAP9+eCMB+BAO +H_0) (Hinshaw et al., 2013) and from PLANCK satellite reported in 2018 (TT,TE,EE+lowE+Lensings +BAO) (Aghanim et al., 2020) are presented in Table 1.

MIXED CHAPLYGIN GAS WITH CANONICAL (PHANTOM) POWER-LAW

In this section, we have separated into two parts. First one, Chaplygin gas with canonical power-law called CGP model. Another one, we have mixed Chaplygin gas with phantom power-law called Chaplygin gas with phantom power-law (CGPP) model. For CGP model, the equation of state parameter from Equation (4) becomes

$$w_{CGP} = -\frac{A}{A + \frac{B}{a_0^6}\left(\frac{t_0}{t}\right)^{6\alpha}}. \tag{10}$$

By using the dust-matter energy density, $\rho_m = \rho_{m,0}(a_0/a)^3$, with canonical power-law, $\rho_m = \rho_{m,0}(t_0/t)^{3\alpha}$, where $\rho_{m,0}$ is a present energy density of dust-matter content in the universe. We can rewrite the effective equation of state parameter, Equation (5), as

$$w_{eff,CGP} = \frac{w_{CGP}}{1 + \rho_{m,0}\left(\frac{t_0}{t}\right)^{3\alpha}\left[A + \frac{B}{a_0^6}\left(\frac{t_0}{t}\right)^{6\alpha}\right]^{-\frac{1}{2}}}. \tag{11}$$

For the CGPP model, the equation of state parameter can be rewritten as

$$w_{CGPP} = -\frac{A}{A + \frac{B}{a_0^6}\left(\frac{t_s - t_0}{t_s - t}\right)^{6\beta}}. \tag{12}$$

With phantom power-law, energy density of dust-matter is

$$\rho_m = \rho_{m,0}\left(\frac{t_s - t_0}{t_s - t}\right)^{3\beta}. \tag{13}$$

The effective equation of state parameter becomes

$$w_{eff,CGPP} = \frac{w_{CGPP}}{1 + \rho_{m,0}\left(\frac{t_s - t_0}{t_s - t}\right)^{3\beta}\left[A + \frac{B}{a_0^6}\left(\frac{t_s - t_0}{t_s - t}\right)^{6\beta}\right]^{-\frac{1}{2}}}. \tag{14}$$

For these two models, we can convert to redshift by using $1 + z = a_0/a$ therefore $t = t_0(1+z)^{-1/\alpha}$ for CGP model and $t_s - t = (t_s - t_0)(1+z)^{-1/\beta}$ for CGPP model. At present, w_0 and $w_{eff,0}$ for both CGP and CGPP models are the same and can be rewritten as

$$w_0 = -\frac{A}{A + B}, \tag{15}$$

and

$$w_{eff,0} = \frac{w_0}{1 + \rho_{m,0}[A + B]^{-\frac{1}{2}}}. \tag{16}$$

These equations depend only on constants *A* and *B*. It is including dust-matter energy density at present, $\rho_{m,0}$, in the Equation (16). Since the values of $\rho_{m,0}$ is very tiny then we can approximate $w_0 \approx w_{eff,0}$.

RESULTS AND DISCUSSIONS

In this section, we have used the derived parameters from Table 1 to calculate the present cosmological parameters from both CGP and CGPP models. The calculation results of both CGP and CGPP models are presented in Table 2. The derived parameters from the CGP model do not match acceleration expansion conditions such as the power exponent α values are $\alpha = 0.9761(6) \pm 0.0154(3)$ (68% CL) by using WMAP9 data and $\alpha = 0.9538(9) \pm 0.0073(0)$ (68% CL) by

Table 2: The prediction parameters from CGP and CGPP models by using WMAP9 (Hinshaw et al., 2013) and PLANCK2018 (Aghanim et al., 2020) datasets. The calculation results from CGP model does not match the observational data but the CGPP model does.

Parameter	**WMAP9** (Hinshaw et al., 2013)	**PLANCK2018** (Aghanim et al., 2020)
α	$0.9761(1) \pm 0.0154(3)$	$0.9538(9) \pm 0.0073(0)$
$q_{0,CGP}$	$0.0244(2) \pm 0.0161(9)$	$0.0483(3) \pm 0.0080(1)$
t_s	$\left(5.248(1)^{+6.056(1)}_{-5.990(1)}\right) \times 10^{18}$ sec	$\left(1.26(4)^{+4.09(7)}_{-4.50(3)}\right) \times 10^{19}$ sec
	$166.2(9)^{+191.8(9)}_{-189.8(0)}$ Gyr	$400.8(1)^{+1299.1(5)}_{-1429.8(9)}$ Gyr
β	$-10.81(1)^{+13.73(1)}_{-13.58(1)}$	$-26.7(5)^{90.0(0)}_{-98.9(0)}$
$q_{0,CGPP}$	$-1.0925(0)^{+0.1174(8)}_{-0.1162(0)}$	$-1.0373(8)^{+0.1257(6)}_{-0.1382(0)}$

using PLANCK2018 data. The condition for accelerating expansion universe is requiring $\alpha > 1$. Another parameter is deceleration parameter q_0 requiring $q_0 < 0$ for accelerating expansion universe and the derived value from CGP model are $q_0 = 0.0244(2) \pm 0.0161(9)$ (68% CL) by using WMAP9 data and $q_0 = 0.0483(3) \pm 0.0080(1)$ (68% CL) by using PLANCK2018 data. The present equation of state parameters, $w_{CGP,0}$, depends only on constants A and B. Therefore, we have evaluated the equation of state parameter by plotting w_{CGP} against time by varying the value of constants A and B as in Figure 1. The plots show that the equation of state w_{CGP} can approach to -1, the observational values of dark energy, as $t \to \infty$ for any values of A and B. At present, the values of $w_{CGP,0}$ can approach to -1 in the case of $A > 0$, $B \geq 0$ and $A \gg B$. In another word, $w_{CGP,0} \to -1$ as A increases and B decreases. For example, $w_{CGP,0} = -0.9901$ if $A = 10$ and $B = 0.1$, $w_{CGP,0} = -0.9999$ if $A = 100$ and $B = 0.01$ and $w_{CGP,0} = -1$ if $B = 0$ with any values of A from both datasets. For $B < 0$ there is a singularity before present as shown in Figure 1 and the results from this figure are the same as we plot $w_{eff,CGP}$ against time. Figure 2 shows the plot of w_{CGP} versus redshift z and the results are the same as shown in Figure 1. By considering all derived parameters from the CGP model, we can conclude that the CGP model must be excluded.

In the case of CGPP model, the derived parameters are shown in Table 2. All results correspond to the accelerating expansion conditions such as the power exponent $\beta < 0$ and the deceleration parameter $q_0 < 0$. The derived values of power exponent are $\beta = -10.81(1)^{+13.73(1)}_{-13.58(1)}$ (68% CL) by using WMAP9 data and $\beta = -26.7(5)^{+90.0(0)}_{-98.9(0)}$ (68% CL) by using PLANCK2018 data. Another one is the deceleration parameter at present $q_0 = -1.0925(0)^{+0.1174(8)}_{-0.1162(0)}$ (68% CL) by using WMAP9 data and $q_0 = -1.0373(8)^{+0.1257(6)}_{-0.1382(0)}$ (68% CL) by using PLANCK2018 data. For the equation of state parameter w_{CGPP} is the same as that of in the CGP model depending only on the constants A and B. Figure 3 shows the plot of the equa-

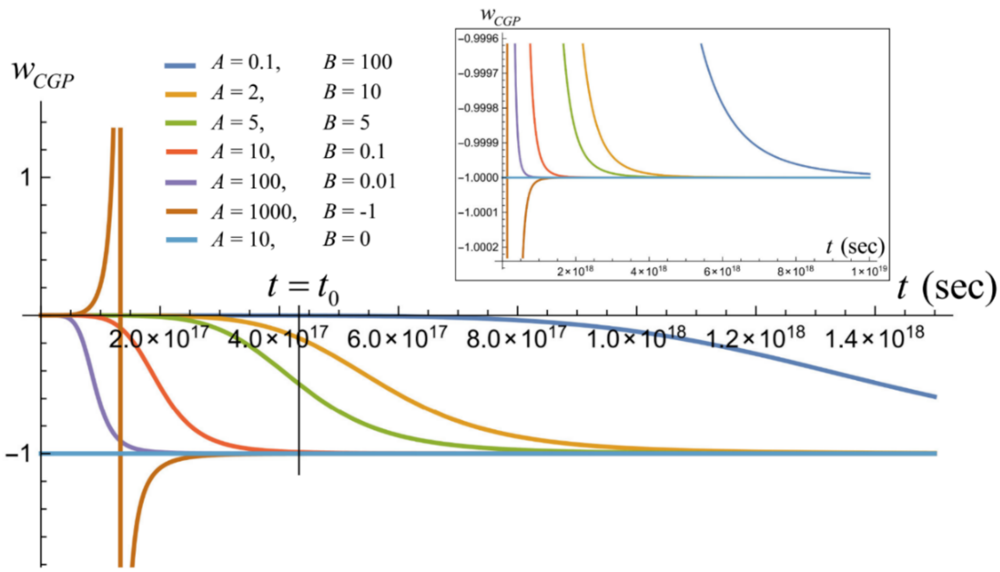

Figure 1: Equation of state parameter of Chaplygin gas with Power-Law model, w_{CGP}, plot against time t by varying constants A and B. At present, it can be approached -1 as A is increasing and B is decreasing with both are positive. **Inset:** As time range increases to 10^{19} seconds, all plots can approach to -1 as $t \to \infty$.

tion of state parameter w_{CGPP} as a function of time. At present, from Equations (15) and (16) we have seen that the results of the equation of state parameter are the same as the results obtaining from CGP model, $w_{CGPP} \to -1$ as $t \to \infty$. But in the CGPP model the equation of state is approaching to -1 faster than that of in the CGP model and there is no singularity behavior has been found from $B < 0$ as $t > 0$. Results of the effective equation of state $w_{eff,CGPP}$ are also the same as that of in the CGP model. For the plot of w_{CGPP} as a function of redshift z, the results are the same as shown in Figure 2. Therefore, the equation of state parameters $w_{CGPP,0}$ can approach to -1 with conditions of $A > 0$, $B \geq 0$ and $A \gg B$.

CONCLUSIONS

In this work, we have investigated the Chaplygin gas model in the scenario of canonical power-law cosmology (CGP model) and phantom power-law cosmology (CGPP model). We have assumed the FLRW universe filled with dust-matter and Chaplygin gas as the dark energy gas. We have considered the universe with dark energy dominated at late time. We have used the WMAP9 (Hinshaw et al., 2013) and PLANCK satellite reported in 2018 (Aghanim et al., 2020) (PLANCK2018) datasets to derive the cosmological parameters from CGP and CGPP models. From those derived parameters in Table 2, we found that the CGP model do not match the conditions of accelerating expansion universe. Therefore, it must be excluded.

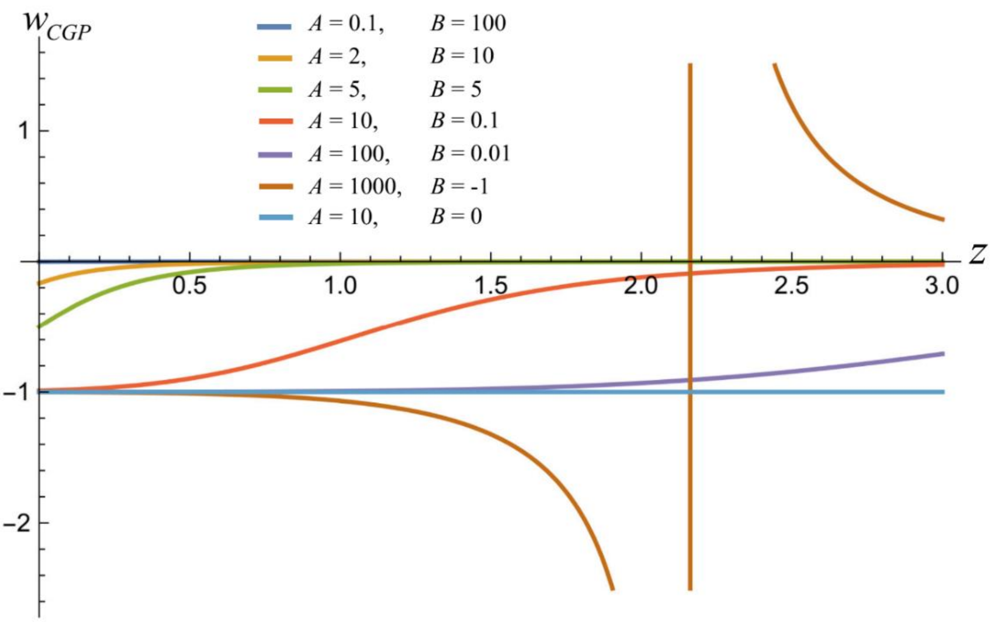

Figure 2: Equation of state parameter of Chaplygin gas with Power-Law model, w_{CGP}, plot against redshift z by varying constants A and B. At present, $z = 0$, $w_{CGP,0}$ can approach to -1 as $A > 0$, $B \geq 0$ and $A \gg B$.

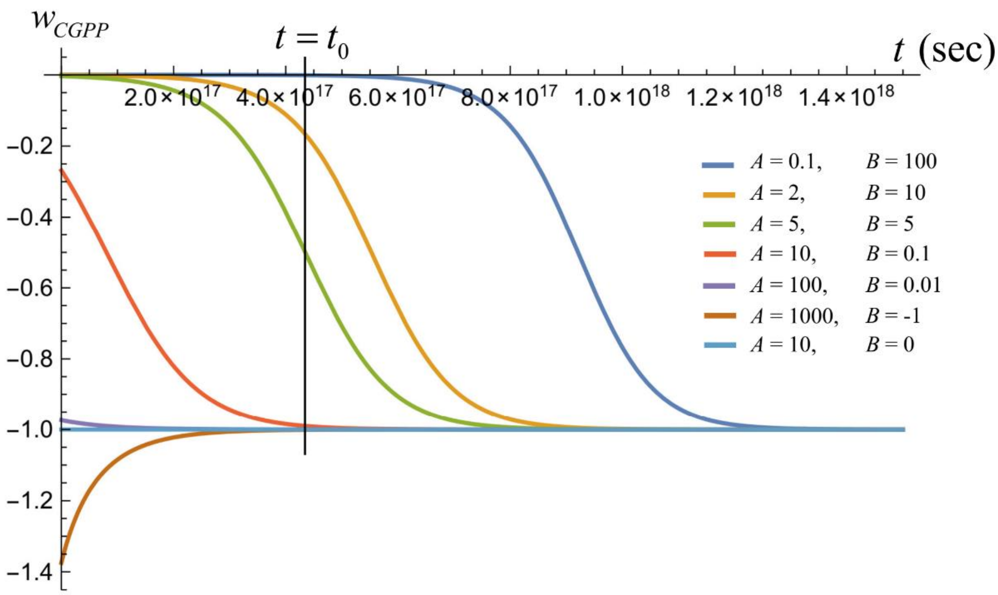

Figure 3: Equation of state parameter of Chaplygin gas with Phantom Power-Law model, w_{CGPP}, plot against time t by varying constants A and B. At present, it can approach to -1 as A is increasing and B is decreasing with positive values of A and any values of B.

In the CGPP model, both the power exponent β and the deceleration parameter q_0 correspond to the conditions of accelerating expansion of the present universe as $\beta < 0$ and $q_0 < 0$. The derived parameters are $\beta = -10.81(1)^{+13.73(1)}_{-13.58(1)}$ (68% CL) by using WMAP9 data, $\beta = -26.7(5)^{+90.0(0)}_{-98.9(0)}$ (68% CL) by using PLANCK2018 data, $q_0 = -1.0925(0)^{+0.1174(8)}_{-0.1162(0)}$ (68% CL) by using WMAP9 data and $q_0 = -1.0373(8)^{+0.1257(6)}_{-0.1382(0)}$ (68% CL) by using PLANCK2018 data. Furthermore, the (effective) equation of state parameters w_{CGP} ($w_{eff,CGP}$) and w_{CGPP} ($w_{eff,CGPP}$) can approach to -1 as $t \to \infty$. At present, the (effective) equation of state parameter is depending only on constants A and B. Its values can approach to -1 with the conditions of $A > 0$, $B \geq 0$ and $A \gg B$ for both CGP and CGPP models, as shown in Figure 1 and Figure 3. For example, $w_{CGPP,0} = -0.9901$ if $A = 10$ and $B = 0.1$, $w_{CGPP,0} = -0.9999$ if $A = 100$ and $B = 0.01$. The equation of state parameter as a function of redshift z, at present $z = 0$, can approach to -1 as $A > 0$, $B \geq 0$ and $A \gg B$ as shown in Figure 2. Therefore, the CGPP model can be used to explain the current accelerating expansion behavior and predict the future fate of the universe.

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