Global stability and optimal control of melioidosis transmission model with hygiene care and treatment

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ABSTRACT

Melioidosis is a bacterial disease and is mostly found in the tropical country especially Southeast Asia, Northern Australia and northeast Thailand. With an increase in number of infected patients and a belief that there is a large number of infections that is underreported, a better understanding and strategy approach in order to reduce the transmission is therefore required. In this study a compartmental model of melioidosis transmission involving hygiene care and treatment is presented. The model is analyzed theoretically and numerically. The basic reproduction number and its sensitivity indices are calculated. Further, by using Pontryagin's Minimum Principle (PMP), the optimal control problem is constructed with two controls. Our results demonstrate that a combination of both hygiene care and treatment controls could largely help reducing the number of exposed and infected individuals and the concentration of bacteria. Therefore, both mentioned controls should be encouraged to reduce overall melioidosis transmission.

Keywords: melioidosis, hygiene care, numerical study, optimal control, treatment

INTRODUCTION

Melioidosis is a life-threatening infectious disease of public health importance in tropics and subtropics particularly in Southeast Asia and Northern Australia. It is caused by the environmental gram-negative bacillus *Burkholderia pseudomallei*, which presents in soil and water (Wiersinga et al., 2012; Limmathurotsakul et al., 2013a). The most common route of melioidosis infection is via direct contact with contaminated soil and water, especially though open wounds on the skin. Humans and animals can also acquire the infection by inhaling dust particles or water droplets or ingesting water that is contaminated. The symptoms can be ranged from acute, high fever, chest pain during breathing to chronic cough with pulmonary infiltration similar to tuberculosis. It is recommended that people with risk factors such as rice farmers, people who have diabetes or immunosuppressive therapy stay indoors during periods of heavy wind and rain, or wear boots and gloves when in direct contact with soil and water and do not consume untreated water (Limmathurotsakul et al., 2013b).

The overall case fatality rate for melioidosis ranges from 14% to 40% and could be as high as 80% if effective antimicrobial drugs are not provided (Hoffmaster et al., 2015). It is estimated to account for 165,000 people who get

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melioidosis infected, from which 89,000 people die per year worldwide (Limmathurotsakul et al., 2016). With this number, it is believed that melioidosis infection is severely underreported in 45 countries, whereas a further 34 countries that have never reported. Most cases found during rainy season or humid months. There is a number of studies confirming the correlation of rainfall and severe weather events and an increase in melioidosis incidence (Liu et al., 2015; Limmathurotsakul et al., 2013b; Mu et al., 2014; Limmathurotsakul et al., 2016; Kaestli et al., 2016). In northeast Thailand, the disease is known to be a major cause of community-acquired septicaemia and is classified as the third most common cause of death from infectious diseases (Limmathurotsakul et al., 2010). The treatment of melioidosis involves antibiotics, however, since the *Burkholderia pseudomallei* are highly drug resistant, the infected ones require prolonged treatment regimens (Dance, 2014).

With the fact above, together with unawareness of people about melioidosis, a number of researches have been performed to better understanding of this disease. For example, the incidence of the melioidosis in Australia (Cheng et al., 2006; Kaestli et al., 2007), Thailand (Cheng et al., 2008; Limmathurotsakul et al., 2010, Limmathurotsakul et al., 2013b; Hinjoy et al., 2018), in Taiwan (Ko et al., 2007; Mu et al., 2014) and in Laos (Rattanavong et al., 2011; Dance et al., 2018). In addition, several mathematical models have been developed to explain the dynamics of the bacterial caused disease, for example cholera (e.g. Cui et al., 2014; Sun et al., 2017) leptospirosis (e.g. Okosun et al., 2016) and typhoid (e.g. Tilahun et al., 2017). To authors' knowledge, there is only one mathematical model of melioidosis transmission has been studied and it is the work by Mahikul et al., 2019. They proposed compartmental model to predict the burden of melioidosis in Thailand containing 8 variables which are susceptible, diabetic susceptible, exposed, symptomatic, asymptomatic, severe, treatment, and recovery. They solved and analyzed their model numerically by using R software and fit their model with raw data by using Markov Chain Monte Carlo (MCMC) method, this was to obtain the structure of melioidosis cases in Thailand and gained parameter values used in the model from this fitting. Their results emphasized on demographic which will lead to estimate the future melioidosis burden in Thailand. However, theoretical analysis including the basic reproduction number, equilibrium point and its stability analysis and control model had not been performed in their model.

Therefore, in this study a compartmental mathematical model which involves bacteria which caused the melioidosis transmission incorporating with hygiene care and treatment control is constructed. Both theoretical and numerical analysis is performed and basic reproduction number is calculated together with its sensitivity indices. Further, the model is extended to optimal control model to determine optimal strategies for controlling the spread of the disease overall.

MODEL FORMULATION

The model in this study involves the effect of hygiene care and treatment on melioidosis transmission. The model contains 4 subclasses which are S is the number of susceptible individuals, E is the number of exposed individuals, I is the number of melioidosis infected individuals, R is the number of recovered individuals, and B is the concentration of bacteria *Burkholderia pseudomallei*. The schematic diagram of the model is shown in Figure 1.

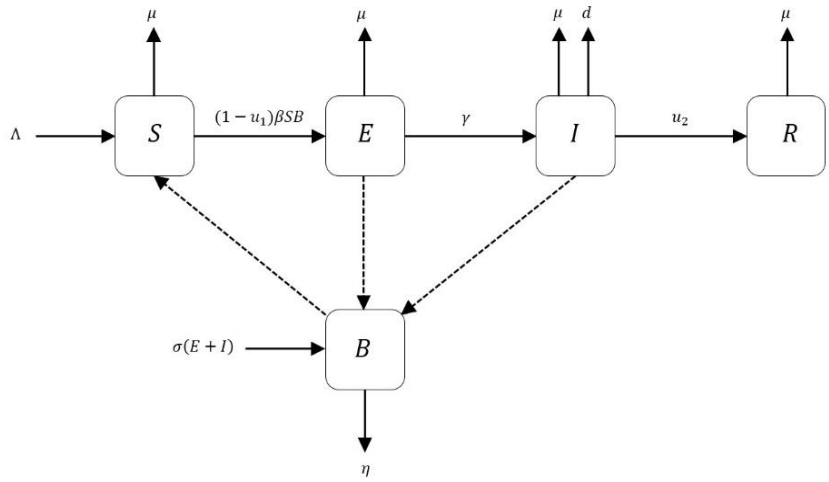

Figure1: A schematic diagram for the transmission of melioidosis.

The model is given by the following system of differential equations.

$$\frac{dS}{dt} = \Lambda - (1-u_1)\beta SB - \mu S \tag{1}$$

$$\frac{dE}{dt} = (1-u_1)\beta SB - (\gamma + \mu)E \tag{2}$$

$$\frac{dI}{dt} = \gamma E - (u_2 + d + \mu)I \tag{3}$$

$$\frac{dR}{dt} = u_2 I - \mu R \tag{4}$$

$$\frac{dB}{dt} = \sigma(E+I) - \eta B, \tag{5}$$

with initial condition

$$S(0) \geq 0,\ E(0) \geq 0,\ I(0) \geq 0,\ R(0) \geq 0,\ B(0) \geq 0\ .$$

The parameters used in this model are defined as Λ is the recruitment rate, β is the transmission rate, μ is the natural death rate, γ is the rate at which exposed individuals become infected, d is the disease induced death rate, σ is an increase rate of bacteria by E and I, η is the bacteria death rate, u_1 is the efficiency of hygiene care control of susceptible individuals, and u_2 is the rate of treatment control for infected individuals.

BOUNDARY OF SOLUTIONS

The boundary of solutions of the system of equation (1) – (5) is determined. By setting $N = S + E + I + R$, we have $N(t) \leq \frac{\Lambda}{\mu} - \left[\frac{\Lambda}{\mu} - N_0\right] e^{-\mu t}$. When $t \to \infty$, we have $N(t) \to \frac{\Lambda}{\mu}$, implying that $0 \leq N(t) \leq \frac{\Lambda}{\mu}$. Hence, the considered region for this model is $\Gamma = \left\{(S, E, I, R) \in \Re_+^4 : N \leq \frac{\Lambda}{\mu}\right\}$. All solutions of this model are bounded and enter the region Γ. Therefore, Γ is a positively invariant. That is every solution of this model remains there for all $t > 0$.

EQUILIBRIUM POINT OF THE MODEL

There are two equilibrium points in this model which are:

i. Disease – free equilibrium point $E_0 = (S_0, E_0, I_0, R_0, B_0) = \left(\frac{\Lambda}{\mu}, 0, 0, 0, 0\right)$.

ii. Endemic – equilibrium point $E_1 = \left(S^*, E^*, I^*, R^*, B^*\right)$ where

$$S^* = \frac{(\gamma + \mu)(u_2 + d + \mu)\eta}{(1 - u_1)\beta\sigma(u_2 + d + \mu + \gamma)},$$

$$E^* = \frac{(u_2 + d + \mu)I^*}{\gamma},$$

$$I^* = \frac{(\gamma + \mu)(u_2 + d + \mu)\gamma\mu\eta + \Lambda\gamma(1 - u_1)\beta\sigma(u_2 + d + \mu + \gamma)}{(1 - u_1)(\gamma + \mu)(u_2 + d + \mu)\beta\sigma(u_2 + d + 2\mu)},$$

$$R^* = \frac{u_2 I^*}{\mu},$$

and $B^* = \frac{\sigma(u_2 + d + \mu + \gamma)I^*}{\gamma\eta}$.

Basic reproduction number (R_0)

The basic reproduction number of this model is calculated by using the next generation method (van den Driessche et al., 2002). From our model, we have

$$R_0 = \frac{(1-u_1)\beta\Lambda\sigma(\gamma+u_2+d+\mu)}{(\gamma+\mu)(u_2+d+\mu)\eta\mu}. \tag{6}$$

LOCAL STABILITY ANALYSIS

The local stability of each equilibrium point within this model is determined from the Jacobian matrix at that equilibrium point of the system of equations (1) - (5). The Jacobian matrix is

$$J(S,E,I,R,B) = \begin{bmatrix} -(1-u_1)\beta B-\mu & 0 & 0 & 0 & 0 \\ (1-u_1)\beta B & -(\gamma+\mu) & 0 & 0 & 0 \\ 0 & \gamma & -(u_2+d+\mu) & 0 & 0 \\ 0 & 0 & u_2 & -\mu & 0 \\ 0 & \sigma & \sigma & 0 & -\eta \end{bmatrix}. \tag{7}$$

Theorem 2.1 (local stability at E_0) If $R_0 < 1$, the disease – free equilibrium point (E_0) is locally asymptotically stable. If $R_0 > 1$, the disease-free equilibrium point (E_0) is unstable.

Proof. The Jacobian matrix of the system of equations (1) - (5) at E_0 is

$$J(\frac{\Lambda}{\mu},0,0,0,0) = \begin{bmatrix} -\mu & 0 & 0 & 0 & -(1-u_1)\beta\dfrac{\Lambda}{\mu} \\ 0 & -(\gamma+\mu) & 0 & 0 & (1-u_1)\beta\dfrac{\Lambda}{\mu} \\ 0 & \gamma & -(u_2+d+\mu) & 0 & 0 \\ 0 & 0 & u_2 & -\mu & 0 \\ 0 & \sigma & \sigma & 0 & -\eta \end{bmatrix}. \tag{8}$$

From Jacobian matrix above, we set $\det(J(E_0)-\lambda I)=0$ to find the eigenvalues, then we obtain

$$(-\mu-\lambda)(-\mu-\lambda)(\lambda^3+(\gamma+2\mu+u_2+d+\eta)\lambda^2+(\eta(\gamma+2\mu+u_2+d)+(\gamma+\mu)(u_2+d+\mu)-$$
$$\frac{(1-u_1)\beta\Lambda\sigma}{\mu})\lambda+\eta(\gamma+\mu)(u_2+d+\mu)-\frac{(1-u_1)\beta\Lambda\sigma(u_2+d+\mu+\gamma)}{\mu})=0.$$

Thus, $\lambda_1=\lambda_2=-\mu<0,$ and

$$\lambda^3+(\gamma+2\mu+u_2+d+\eta)\lambda^2+(\eta(\gamma+2\mu+u_2+d)+(\gamma+\mu)(u_2+d+\mu)$$
$$-\frac{(1-u_1)\beta\Lambda\sigma}{\mu})\lambda+\eta(\gamma+\mu)(u_2+d+\mu)-\frac{(1-u_1)\beta\Lambda\sigma(u_2+d+\mu+\gamma)}{\mu}=0$$ which is

considered in the form of $\lambda^3+a_1\lambda^2+a_2\lambda+a_3=0.$ Therefore, $a_1=\gamma+2\mu+u_2+d+\eta,$

$a_2=\eta(\gamma+2\mu+u_2+d)+(\gamma+\mu)(u_2+d+\mu)-\frac{(1-u_1)\beta\Lambda\sigma}{\mu}$ and

$a_3=(\gamma+\mu)(u_2+d+\mu)\eta(1-R_0).$

It is clearly seen that $a_1>0$ and $a_3>0$ when $R_0<1$. Furthermore, by our calculation which is omitted here we also obtain that $a_1a_2>a_3$ when $R_0<1$. Therefore, by Routh-Hurwitz Criterion the disease – free equilibrium point is locally asymptotically stable when $R_0<1$.

Theorem 2.2 (local stability at E_1) When $R_0>1$, the endemic equilibrium point (E_1) is stable if it satisfies the Routh-Hurwitz criterion.

Proof. Considering from Jacobian matrix of endemic equilibrium point, we have

$$J(S^*,E^*,I^*,R^*,B^*)=\begin{bmatrix} -(1-u_1)\beta B^*-\mu & 0 & 0 & 0 & -(1-u_1)\beta S^* \\ (1-u_1)\beta B^* & -(\gamma+\mu) & 0 & 0 & (1-u_1)\beta S^* \\ 0 & \gamma & -(u_2+d+\mu) & 0 & 0 \\ 0 & 0 & u_2 & -\mu & 0 \\ 0 & \sigma & \sigma & 0 & -\eta \end{bmatrix}. \quad (9)$$

Setting $\det(J(E_1)-\lambda I)=0,$ we have the first eigenvalue $\lambda=-\mu<0$. The rest of characteristic equation is considered in the form of $\lambda^4+a_1\lambda^3+a_2\lambda^2+a_3\lambda+a_4=0,$ where

$$a_1=(\gamma+\mu)+(u_2+d+\mu)+\eta+\mu+(1-u_1)\beta B^*>0,$$
$$a_2=(\gamma+\mu)\eta-\sigma(1-u_1)\beta S^*+(u_2+d+\mu)(\gamma+\mu-\eta)+((1-u_1)\beta B^*+\mu)(u_2+d+3\mu+\gamma),$$

$$a_3=(\gamma+\mu)(u_2+d+\mu)\eta-\sigma(1-u_1)(u_2+d+\mu)\eta S^*+((1-u_1)\beta B^*+\mu)((\gamma+\mu)\mu-$$
$$\sigma(1-u_1)\beta S^*+(u_2+d+\mu)(\gamma+\mu+\eta))+(1-u_1)\beta S^*\sigma((1-u_1)\beta B^*-\gamma),$$
$$a_4=((1-u_1)\beta B^*+\mu)((\gamma+\mu)(u_2+d+\mu)\eta-\sigma(1-u_1)(u_1+d+\mu)\beta S^*)$$
$$+(1-u_1)\beta S^*\sigma((1-u_1)(u_2+d+\mu)\beta B^*-\gamma\mu).$$

By using Routh-Hurwitz Criterion for $n=4,$ the endemic equilibrium point is stable if $a_3>0,a_4>0$ and $a_1a_2a_3>a_3^2+a_1^2a_4.$

GLOBAL STABILITY ANALYSIS

Theorem 2.3 (global stability at E_0) *If* $R_0 < 1$, then E_0 is globally asymptotically stable.

Proof. Let the Lyapunov function be as follows:

$$L = \left(\frac{\sigma\gamma + \sigma(u_2 + d + \mu)}{(\gamma + \mu)(u_2 + d + \mu)\eta} \right) E + \left(\frac{\sigma}{(u_2 + d + \mu)\eta} \right) I + \left(\frac{1}{\eta} \right) B$$

Calculate the derivative of L together with the use of boundary of solutions, we obtain

$$L' = B\left(\frac{(1-u_1)\beta S\sigma(\gamma + u_2 + d + \mu)}{(\gamma + \mu)(u_2 + d + \mu)\eta} - 1 \right) \le B\left(\frac{(1-u_1)\beta\Lambda\sigma(\gamma + u_2 + d + \mu)}{(\gamma + \mu)(u_2 + d + \mu)\eta\mu} - 1 \right) = B(R_0 - 1)$$

When $R_0 < 1$, we obtain that $L' < 0$ and $L' = 0$ when $E = I = B = 0$. Hence, E_0 is globally asymptotically stable when $R_0 < 1$.

Theorem 2.4 (global stability at E_1) The endemic equilibrium point is globally stable when $R_0 > 1$.

Proof. We have used the geometric approach by Li and Muldowney (1993) to prove this Theorem. Due to the long calculation, it is omitted here.

SENSITIVITY ANALYSIS

The sensitivity indices are calculated by using the normalized forward sensitivity index technique (Ngoteya and Gyekye, 2015: Samsuzzoha et al., 2013). With the use of parameters value in Table 2, the sensitivity indices are given in Table 1. The results show that to reduce the value of R_0, we could try to increase the value of $u_1, \eta, u_2, \mu, \gamma$ and d, respectively, whereas we could try to reduce the value of Λ, β and σ, respectively.

Table 1: Numerical values of sensitivity indices of R_0

Parameters	Index at Parameter Value	Sign
Λ	+1.0000	positive
β	+1.0000	positive
σ	+1.0000	positive
d	-0.0341	negative
γ	-0.4708	negative
μ	-0.4716	negative
u_2	-0.4943	negative
η	-1.0000	negative
u_1	-1.0000	negative

NUMERICAL SIMULATION

In this section, the system of equations (1) – (5) is numerically solved by using Euler's method. The parameters within this model are chosen as the appropriate and are shown in Table 2. The numerical results are shown in Figure 2 – 3.

Table 2: Parameters values used in numerical study

Parameter	Description	Value	Reference
Λ	The recruitment rate	1.000000 per week	Assume
β	The transmission rate	0.004000 per week	Khan, M.A. et al., 2014
μ	The natural death rate	0.000296 per week	Pongsumpun, P. 2017
γ	The rate which exposed individuals become infected individuals	0.600000 per week	Tilahun G.T. et al., 2017
d	The disease induced death rate	0.034500 per week	The bureau of epidemiology, 2016
σ	An increase rate of bacteri by E and I	0.800000 per week	Tilahun, G.T. et al., 2017
η	The bacteria death rate	0.500000 per week	Assume
u_1	The efficiency of hygiene care control of susceptible human	0.500000 per week	Variable
u_2	The rate of treatment control for infected individuals	0.500000 per week	Variable

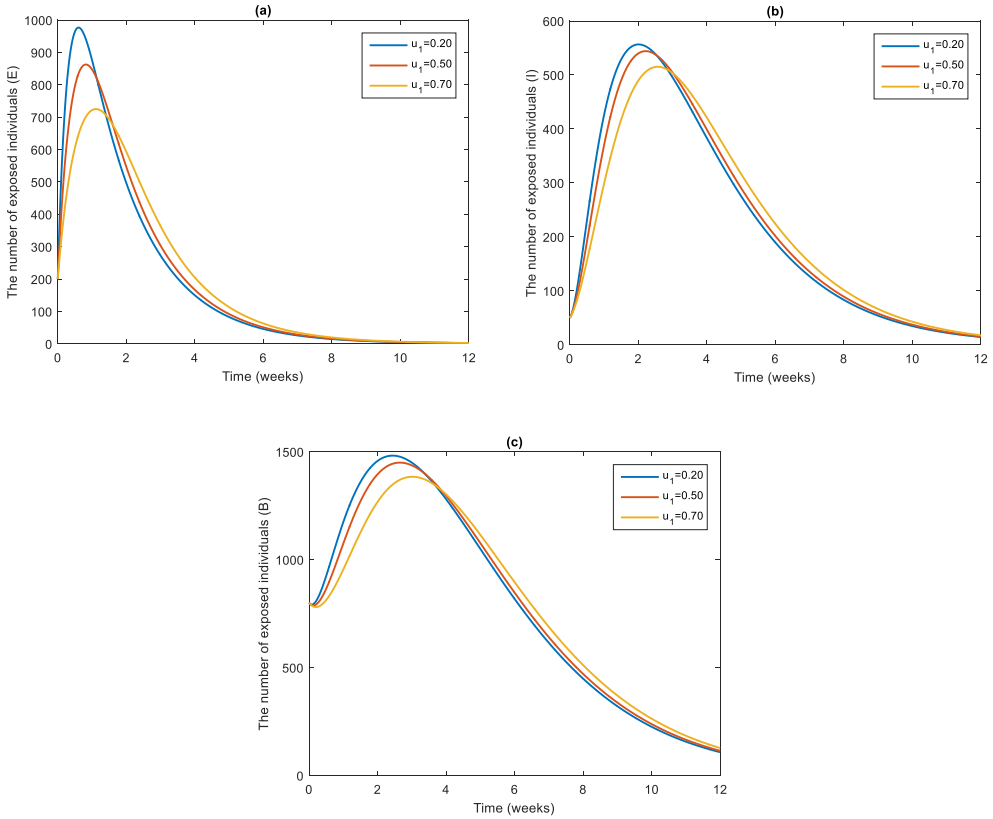

Figure 2: Numerical solution of system of equations (1) - (5) obtained using parameters: $\Lambda = 1,\ \beta = 0.004000, \mu = 0.000296, \gamma = 0.600000, d = 0.034500, \sigma = 0.8000000, \eta = 0.500000$ where (a) is the population of exposed individuals (E), (b) is the population of melioidosis infected individuals (I) and (c) is the concentration of bacteria *Burkholderia pseudomallei* (B), when u_1 varies.

Figure 2 shows that when the efficiency of hygiene care control (u_1) increases, the number of exposed individuals, infected individuals and the concentration of bacteria decrease, respectively. This indicates a good impact of hygiene care in reducing the melioidosis transmission.

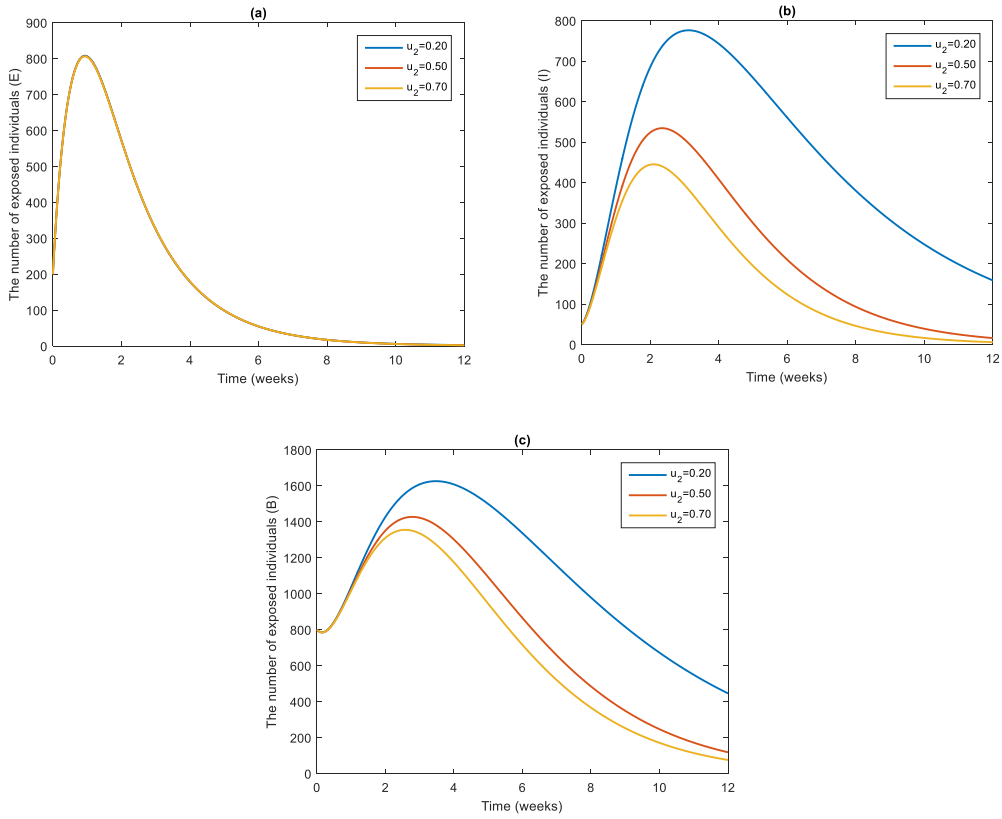

Figure 3: Numerical solution of system of equations (1) - (5) obtained using parameters: $\Lambda = 1,\ \beta = 0.004000, \mu = 0.000296, \gamma = 0.600000, d = 0.034500, \sigma = 0.8000000, \eta = 0.500000$ where (a) is the population of exposed individuals (E), (b) is the population of melioidosis infected individuals (I) and (c) is the concentration of bacteria *Burkholderia pseudomallei* (B), when u_2 varies.

Figure 3 shows that when the rate of treatment control (u_2) increases, the number of exposed individuals is not changed whereas the number of infected individuals and the concentration of bacteria dramatically decrease and tend to reach lower equilibrium value. Therefore, treatment of infected individuals should be encouraged.

OPTIMAL CONTRAOL

In this section, we apply the optimal control in our model i.e. the system of equation (1) - (5). For the control problem, we consider two control variables, i.e. u_1 represents the rate of hygiene care control for susceptible individuals and u_2 represents the rate of treatment control for infected individuals. Assume further that ϕ is the rate of normal treatment for patient equals to 0.5 per week. A diagram of this control model is shown in Figure 4.

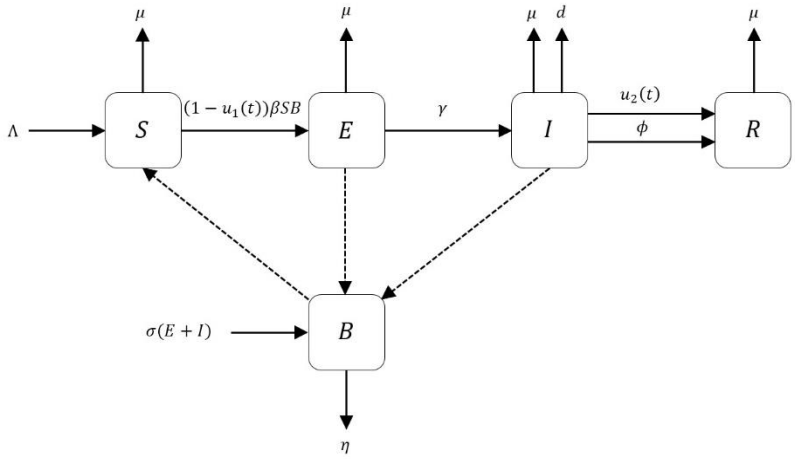

Figure 4: Diagram of the optimal control model of melioidosis.

This model can be written as the system of the equations as follows:

$$\frac{dS}{dt} = \Lambda - (1-u_1(t))\beta SB - \mu S \tag{10}$$

$$\frac{dE}{dt} = (1-u_1(t))\beta SB - (\gamma + \mu)E \tag{11}$$

$$\frac{dI}{dt} = \gamma E - (u_2(t) + d + \mu + \phi)I \tag{12}$$

$$\frac{dR}{dt} = (u_2(t) + \phi)I - \mu R \tag{13}$$

$$\frac{dB}{dt} = \sigma(E + I) - \eta B. \tag{14}$$

All parameters definitions are the same as the system of equations (1) – (5). The model is analyzes basing on the theory of Pontryagin et al. (1986). For the optimal control model, the objective of the model is given by:

$$J(u_1,u_2)=\min\int_0^T\left[M_1E+M_2I+\frac{1}{2}\left(M_3u_1^2(t)+M_4u_2^2(t)\right)\right]dt, \tag{15}$$

with initial condition $S(0)\geq 0, E(0)\geq 0, I(0)\geq 0, R(0)\geq 0$ and $B(0)\geq 0$. The constants M_1, M_2, M_3 and M_4 are weight constants and the terms $M_3u_1^2(t)$ and $M_4u_2^2(t)$ represent the costs associated with hygiene care control and treatment control for melioidosis infected, respectively.

We can determine an optimal solution of this optimal control problem by considering the Lagrangian and the Hamiltonian for the problem. The Lagrangian of the optimal control problem is given by

$$f(E,I,u_1,u_2)=M_1E+M_2I+\frac{1}{2}\left(M_3u_1^2(t)+M_4u_2^2(t)\right). \tag{16}$$

Applying Pontryagin's Minimum Principle (PMP), we form the Hamiltonian and derive the optimality system:

$$\begin{aligned}H=&M_1E+M_2I+\frac{1}{2}\left(M_3u_1^2(t)+M_4u_2^2(t)\right)+\lambda_S\left[\Lambda-(1-u_1(t))\beta SB-\mu S\right]\\&+\lambda_E\left[(1-u_1(t))\beta SB-(\gamma+\mu)E\right]+\lambda_I\left[\gamma E-(u_2(t)+d+\mu+\phi)I\right]\\&+\lambda_R\left[(u_2(t)+\phi)I-\mu R\right]+\lambda_B\left[\sigma(E+I)-\eta B\right],\end{aligned} \tag{17}$$

where $\lambda_S, \lambda_E, \lambda_I, \lambda_R$ and λ_B are the adjoint functions associated with the state equations for S, E, I, R and B, respectively. The adjoint equations by setting $S(t)=\tilde{S}, E(t)=\tilde{E}, I(t)=\tilde{I}, R(t)=\tilde{R}$ and $B(t)=\tilde{B}$, are

$$\begin{aligned}\lambda_S^{'}&=-\frac{\partial H}{\partial S}=-\left[-\lambda_S\left[(1-u_1(t))\beta\tilde{B}+\mu\right]+\lambda_E\left[(1-u_1(t))\beta\tilde{B}\right]\right],\\\lambda_E^{'}&=-\frac{\partial H}{\partial E}=-\left[M_1-\lambda_E\left[\gamma+\mu\right]+\lambda_I\gamma+\lambda_B\sigma\right],\\\lambda_I^{'}&=-\frac{\partial H}{\partial I}=-\left[M_2-\lambda_I\left[u_2(t)+d+\mu+\phi\right]+\lambda_R\left[u_2(t)+\phi\right]+\lambda_B\sigma\right],\\\lambda_R^{'}&=-\frac{\partial H}{\partial R}=-\left[-\lambda_R\mu\right],\\\lambda_B^{'}&=-\frac{\partial H}{\partial B}=-\left[-\lambda_S\left[(1-u_1(t))\beta\tilde{B}\right]+\lambda_E\left[(1-u_1(t))\beta\tilde{B}\right]-\lambda_B\eta\right].\end{aligned} \tag{18}$$

The optimal control variables $u_1^*(t)$ and $u_2^*(t)$ are given by

$$u_1^*(t) = \max\left\{0, \min\left\{\frac{(\lambda_E - \lambda_S)\beta\tilde{S}\tilde{B}}{M_3}, u_{\max}\right\}\right\},$$
$$u_2^*(t) = \max\left\{0, \min\left\{\frac{(\lambda_I - \lambda_R)\tilde{I}}{M_4}, u_{\max}\right\}\right\}, \quad (19)$$

where it is subject to the constraint $0 \le u_1(t) \le u_{\max}$ and $0 \le u_2(t) \le u_{\max}$. The characterization of the optimal control variables $u_1^*(t)$ and $u_2^*(t)$ are given by: $u_1(t) = \frac{(\lambda_E - \lambda_S)\beta\tilde{S}\tilde{B}}{M_3}$, and $u_2(t) = \frac{(\lambda_I - \lambda_R)\tilde{I}}{M_4}$.

Numerical simulation of optimal control

The numerical results of this optimal control model are shown in Figure 5 and 6. We use the forward – backward sweep method and solved the optimality system numerically using Euler's method.

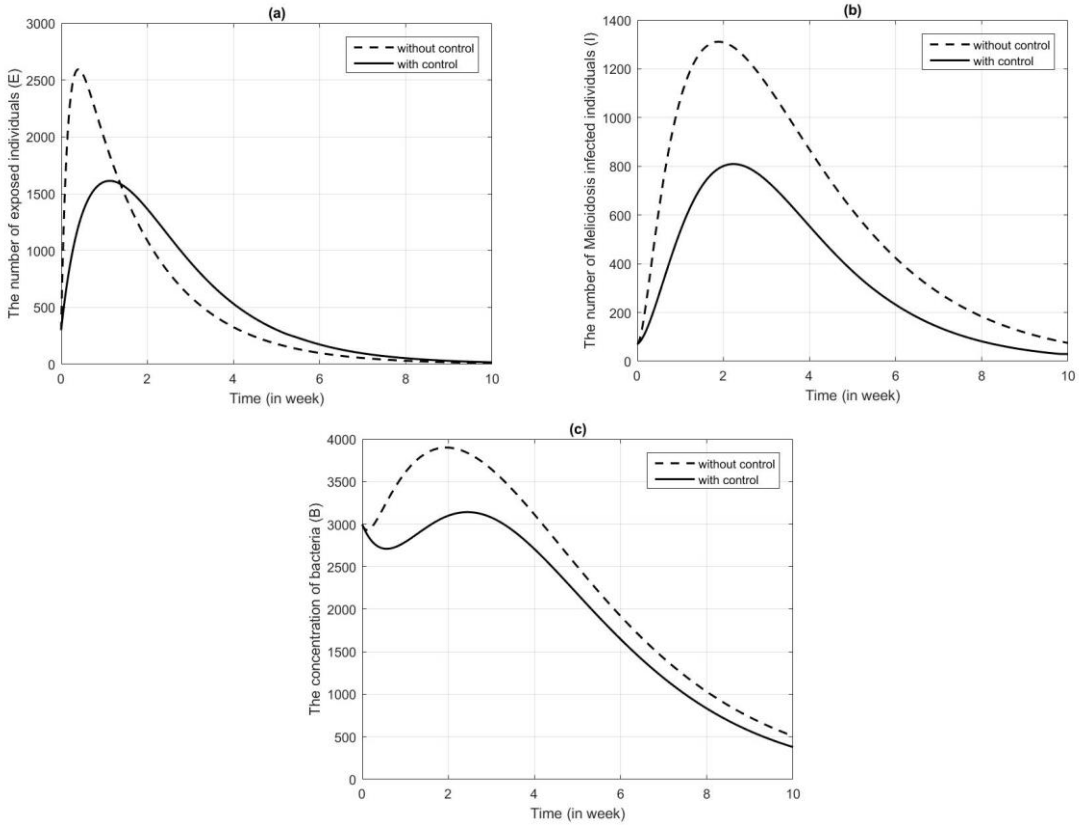

Figure 5: Numerical simulation of (a) population of E, (b) population of I and (c) the concentrate of B in the case of with and without controls.

Figure 5 (a) shows a large decrease in the number of exposed individuals at the peak in the controls case although between 2nd to 6th weeks, the number of exposed individuals is slightly more than in no controls case. Figure 5 (b) and (c) demonstrates a dramatic reduction in the number of infected individuals and a clear decrease in the concentration of bacteria in the controls case, respectively.

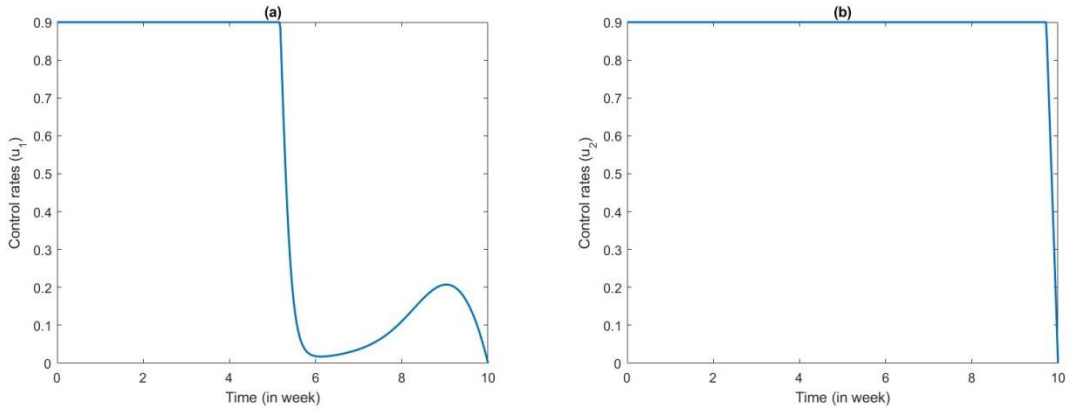

Figure 6: Dynamic of (a) hygiene care control (u_1) and (b) treatment control u_2.

Figure 6 (a) shows that we may keep the hygiene care control for susceptible individuals (u_1) at the maximum rate of 90% from the beginning until the 5th week and may drop the control to less than 10% in the 6th week, after that we are required to increase the rate of the control gradually to reach approximately 20% in the 9th week and then can drop it again. However, Figure 6 (b) indicates that we are required to keep the treatment control for infected individuals (u_2) at the maximum rate almost all the time in order to keep the number of exposed and infected individuals and the concentration of bacteria in the low level as Figure 5.

CONCLUSIONS

In this study, a compartmental model of melioidosis transmission involving hygiene care and treatment is presented. Two main equilibrium points (i.e. disease-free and endemic) are obtained. The basic reproduction number is $R_0 = \frac{(1-u_1)\beta\Lambda\sigma(\gamma+u_2+d+\mu)}{(\gamma+\mu)(u_2+d+\mu)\eta\mu}$, where it becomes a threshold for equilibria stability i.e. when $R_0 < 1$ the disease-free equilibrium point is both locally and globally stable and unstable otherwise. When $R_0 > 1$, the endemic equilibrium point is globally asymptotically stable. Our numerical results demonstrate that the efficiency of hygiene care control of susceptible individuals u_1 shows the impact in reducing the number of exposed individuals and slightly reducing the number of infected individuals and the concentration of bacteria. On the other hand, the rate of treatment control for infected individuals u_2 gives a big impact in reducing the number of infected individuals and the concentration of bacteria dramatically whereas it does not change the number of exposed individuals. These results are confirmed by the sensitivity analysis results. Finally, optimal control model shows that with the combination of both control variables $u_1(t)$ and $u_2(t)$, the number of exposed and infected individuals and the concentration of bacteria are largely reduced. Therefore, both hygiene care and treatment controls should be encouraged as a promising approach to reduce the melioidosis transmission overall.

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