High Speed Video Technique Analysis of an Object Moving on an Inclined Plane

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ABSTRACT

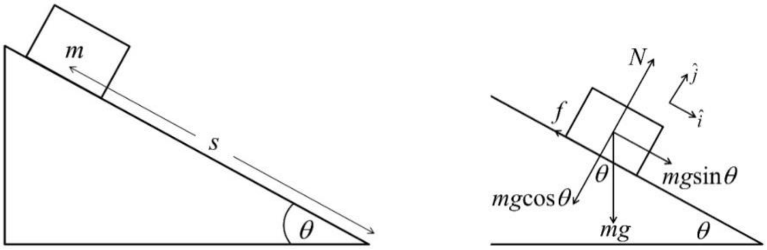
A high-speed (HS) video technique was used to record and analyze the motion of an object moving on an inclined plane. The coefficient of static and kinetic friction between the wooden plane and the object, the acceleration of the wooden plate, released at the different angles and, the total mechanical energy of the system were investigated. The research found that the wooden plate starts moving at the critical angle of around 25.66 ± 0.23 degrees and the static and kinetic friction coefficient values were 0.48 and 0.40, respectively. The clear understanding of students and teachers of this or other physical phenomena can be further performed via this visualization technique.

Keywords: high-speed video, inclined plane, coefficient, acceleration, mechanical energy

INTRODUCTION

An understanding of classical mechanics as well as a wide range of physics phenomena, such as motion of mass, force, work and energy, momentum, torque, and angular momentum, is fundamental and necessary. However, previous studies found most students' disconnection between physics and their real-life experiences (Redish, Saul and Steinberg, 1998; Elby, 2001). They often see the physics done in class as concerning only specially prepared situations, unless the material is an example that they see as relevant or directly related to theirs experience (Redish, Saul and Steinberg, 1998). This might cause a serious problem in terms of students' attitude in learning physics because the learning tends to be more effective and robust when linked to real and personal experiences. Therefore, many physics education researches attempted to implement real life examples into teaching introductory physics, so that students would develop a positive outlook on physics (Esclada and Zollman, 1997; Elby, 2001; Wattanakasiwich and Poonyawatpornkul, 2012). Nowadays, high-speed digital video cameras and video analysis software have been increasing in popularity among physicists and physics teachers to record mechanics situations (Heck and Uylings, 2009; Kaewsutthi and Wattanakasiwich, 2011), especially in motion cases such as exploding balloons, deformed balls, strange reflections, and breaking rods (Vollmer and Mollmann, 2011), the effects of drag force on falling spheres (Owen and William, 2005), free falling object and simple pendulum (Glawtanong, Ritphan, Sirisathitkul, Yaiprasert, and Sirisathitkul, 2011), rolling cylinders (Phommarach, Wattanakasiwich and Johnston, 2012), damped harmonic motion (Poonyawatpornkul and Wattanakasiwich, 2013), rolling disc (Poonyawatpornkul and Wattanakasiwich, 2015). These help teachers and students to see actual motions. Then they can link the real world situations with physics problems in the classroom (Brown and Cox, 2009; Wee, Tan, Leong, and Tan, 2015). Therefore, a high-speed video recorder is aimed to be used in mechanics teaching on an object moving on an inclined plane, which includes the coefficient of static and kinetic friction between the wooden plane and the plate evaluation, the acceleration of wooden plate released from different plate's angles and the total mechanical energy of the system.

There are theories involved in this work, as shown in the following. An object, having mass m, rests on an inclined plane, ranging s from horizontal plane as shown in figure 1(a). The inclined angle is increased until the object starts to move at the critical angle θ_C, when the coefficient of static friction (μ_s) between the surface and the object occurs.



(a) (b)

Figure 1 (a) Rest object on an inclined plane. (b) All external forces act on the object.

Consider a coordinate system with the positive $x-$direction pointing down the inclined plane (figure 1(b)). The normal force is given by $\vec{\mathbf{N}} = N\,\hat{\mathbf{j}}$. The magnitude N of the normal force is determined from Newton's Second Law applied to the normal direction to the inclined plane as equation (1).

$$N - mg\cos\theta = 0 \tag{1}$$

The static frictional force is then, $\vec{f}_s = -mg\sin\theta\,\hat{\mathbf{i}}$. The magnitude f_s is

$$f_s = mg\sin\theta = \frac{N}{\cos\theta}\sin\theta = N\tan\theta \tag{2}$$

When the object starts moving, the static frictional force has reached $\mu_s N$. Then μ_s becomes:

$$\mu_s = \tan\theta_C \tag{3}$$

After the object starts moving at $\theta \geq \theta_C$, it accelerates down the inclined plane. Kinetic frictional force (f_k) occurs whose magnitude f_k is

$$f_k = \mu_k N = \mu_k mg\cos\theta \tag{4}$$

The relation between acceleration (a) of the object and μ_k can be determined from the Newton's Second Law, applied on the $x-$direction, as shown in the following equations.

$$ma = mg\sin\theta - \mu_k mg\cos\theta \tag{5}$$

Thus,
$$a = g\sin\theta - \mu_k g\cos\theta \tag{6}$$

The work done by the friction force, a non-conservative force, is then

$$W_{\text{friction}} = \int_i^f \vec{f}_k \cdot d\vec{\mathbf{s}} = -\mu_k mg\cos\theta\, s \tag{7}$$

In this situation, there are both conservative and non-conservative forces acting on the object. Thus the net work done is,

$$W_{\text{net}} = W_C + W_{NC} = \Delta KE \tag{8}$$

where W_C is work done by conservative force (by the definition of PE, $W_C = -\Delta PE$), and W_{NC} is work done by non-conservative force ($W_{NC} = W_{\text{friction}}$). The equation (8) becomes:

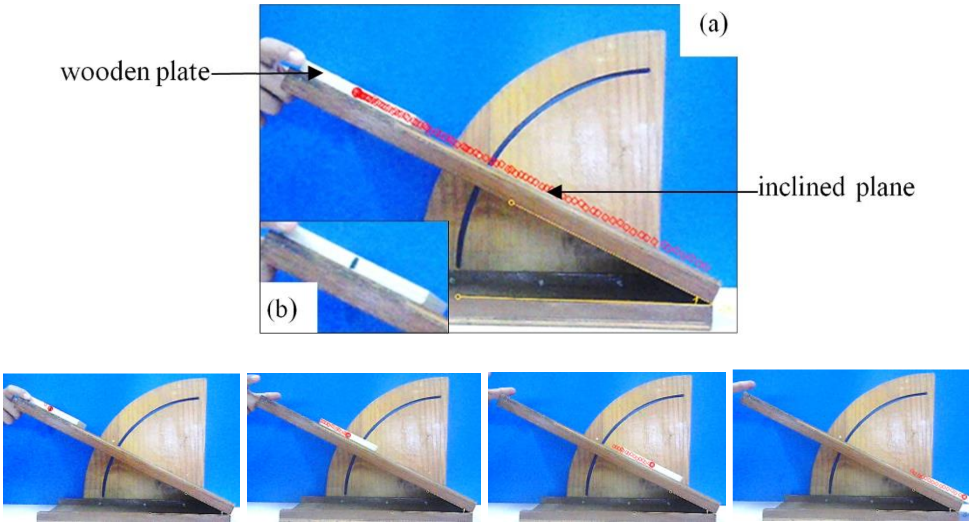
$$\Delta KE = -\Delta PE + W_{\text{friction}}$$

or
$$W_{\text{friction}} = \Delta KE + \Delta PE = \Delta E_{mech} = E_{final} - E_{initial} \tag{9}$$

where ΔE_{mech} is the different mechanical energy. If the work on the object is conservative force, W_{friction} will be zero. Then its mechanical energy does not change ($E_{final} = E_{initial}$).

MATERIALS AND METHODS

An object moving on an inclined plane and its characteristics were studied via a high-speed video technique. Main equipment used were (1) the high speed video camera (Casio EX-FH100), having high frame rates at 120, 240 fps etc. for recording the motion on the plane, (2) the tracker video analysis software (Tracker, the free software tool) (Brown, 2012), which was used to analyze the video of motions. The positions x, y and times of the motions can be obtained and then velocity and acceleration of the motion can be further analyzed, and (3) the spreadsheet software. The apparatus and supplies of this experiment are shown in figure 2.



t = 0.00 s t = 0.40 s t = 0.60 s t = 0.76 s

(c)

Figure 2 Using tracker to analyze an object moving on an inclined plane. (a) An actual setup, (b) an enlarged picture of the monitoring point on the wooden plate and (c) the wooden plate movement series at 0.00, 0.40, 0.60 and 0.76 seconds.

A wooden plate, used as the object moving on an inclined adjustable angle plane, having a mass of 46.3 g was used. The motions were captured by the HS-video at 120 frames per second. A black marker on the wooden plate was set as a pointed monitoring in Tracker, as shown in figure 2(b). After tracking the marker, the movements of the wooden plate at various time (t) are shown in figure 2(c). Three main study areas include (1) the static friction coefficient (determined from the angle that the wooden plate starts moving, θ_C), (2) acceleration and kinetic friction coefficients related to different released angles from 27 to 65 degrees, and (3) total energy of each case.

RESULTS AND DISCUSSION

Coefficient of static friction

We determined the critical angle that the wooden plate started moving down the inclined plane. The critical angle was obtained from “Protractor”, which is a tool in Tracker, as shown in figure 3. We found the average critical angle which was 25.66 ± 0.23 degrees. Therefore, the coefficient of static friction between the wooden plate and the inclined plane, determined from equation (3), was 0.48 ± 0.03.

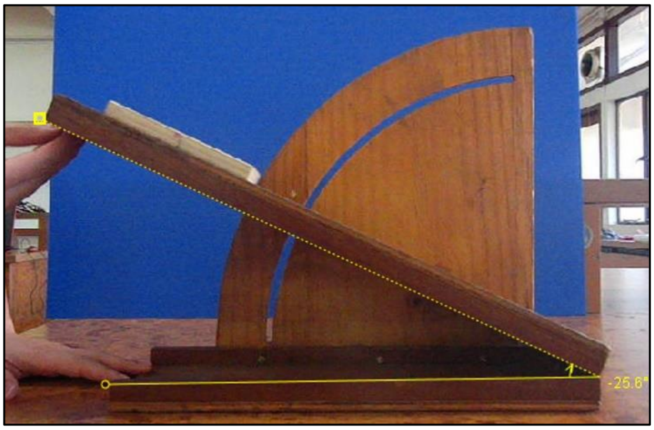
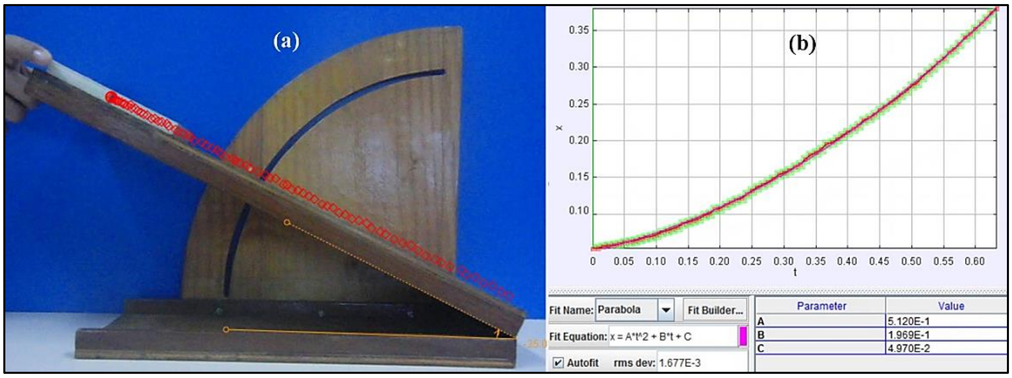


Figure 3 “Protractor” tool in Tracker was used in the angle determination.

Accelerations and coefficient of kinetic friction

The HS-video technique was used to capture the position of the wooden plate moving on the plane at 9 different angle values of 27, 30, 35, 40, 45, 50, 55, 60, and 65 degrees. The wooden plate position change (figure 4(a)) was fitted as a parabola curve with time, written as $y = At^2 + Bt + C$, then the parameters A, b, and c were evaluated (figure 4(b)). The acceleration (a) of the wooden plate was determined from fitting and was then compared with the equation of $y = ut + (1/2)at^2$. The coefficient of kinetic friction between the wooden plate and the inclined plane was then analyzed using equation (6).



Fit Name: Parabola

Fit Equation: x = A*t^2 + B*t + C

☑ Autofit rms dev: 1.677E-3

Parameter	Value
A	5.120E-1
B	1.969E-1
C	4.970E-2

Figure 4 Using a Data tool application in Tracker to determine an acceleration. (a) tracking the wooden plate position change with time and (b) the curve fitting of position versus time graph with $y = At^2 + Bt + C$

The analyzed data result is shown in figure 5, showing the acceleration and coefficient of kinetic friction on y-axis, related to the releasing angle lying on the x-axis. The acceleration, marked as ◇, indicates that it is directly proportional to the angle of an inclined plane. It is clearly seen that the wooden plate moves faster as the releasing angle increases. The kinetic friction coefficients, marked as ◆, are nearly consistent as the inclined angle increases, showing an independence of the releasing angle. These coefficient values can be averaged as 0.40 ± 0.05, given an agreement with the literature coefficient values of wooden surfaces (Engineershandbook.com., 2016). The relation between the frictional force $(\mu mg\cos\theta)$ and the gravitational force acting on the plate on the inclined plane $(mg\sin\theta)$ is shown in figure 6.

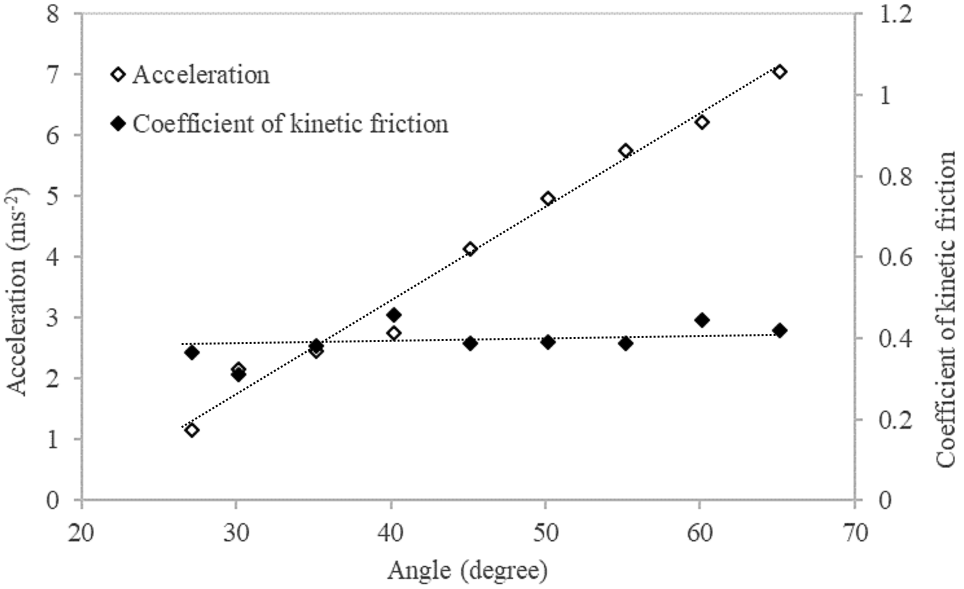


Figure 5 The relationship between accelerations, coefficient of kinetic friction, and releasing angle of the object on the inclined plane.

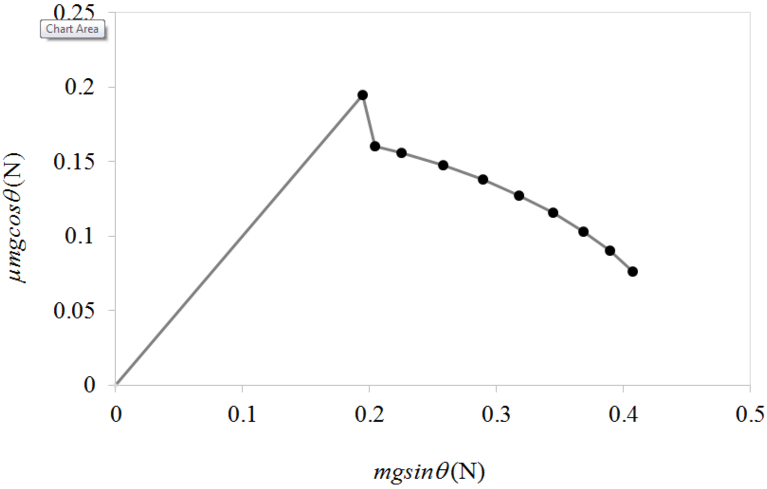


Figure 6 The relation between the frictional force **(** $\mu mg\cos\theta$) and the gravitational force acting on the plate on the inclined plane ($mg\sin\theta$).

Furthers analysis on the relation of the fictional force, $\mu mg\cos\theta$, and the magnitude of $mg\sin\theta$ was determined as shown in Figure 6, presented two forces acting on the wooden plate in motional direction ($x-$direction). The fictional force acts in negative direction of moving, while the other of $mg\sin\theta$ points in the moving direction. Three regions of the motion can be classified. During the earliest stage of the angles of 0 to 25 degrees, no movement of the wooden plate is observed. This means that $\mu mg\cos\theta$ is equal to $mg\sin\theta$. The second stage, the wooden plate starts moving at the critical angle of around 25.66 ± 0.23 degrees. At this angle, the maximum static frictional force was occurred at about 0.195 N, as shown in figure 6. The last stage defined as the releasing angle is higher than 25.66 ± 0.23 degrees and $mg\sin\theta$ was increased. This means that while $mg\sin\theta$ increases, as the greater incline angle, the higher tractive force will be found. This tractive force makes lower value of frictional force component, $mg\cos\theta$. Therefore, f_k drops while an incline angle increases or an easier sliding down can be occurred. These investigated results are in agreement with the theoretical explanation shown in equation (4).

Energy consideration

Based on the work-energy theorem, the wooden plate marker center was monitored for its energy conservation. Positions of this wooden plate at each time frame were obtained with Tracker and imported to a standard spreadsheet. They were used to determine gravitational potential energy, kinetic energy, total mechanical energy and frictional work of the wooden plate at a released angle of 30°, showing in figure 7. This figure illustrates the total mechanical energy (marker as ▲), the summation of potential energy (indicated as ○) and kinetic energy (indicated as Clearly decreasing of the total mechanical energy values (▲) with times, according to the frictional work done (□) on the system, was found. This indicates non conservation of the total energy. However, the additional of the work done by frictional force to those energies shows a consistency in total mechanical energy (marker as ◆) at the approximated value of 0.063±0.002 J. This agrees that the work done by a non-conservative force causes the energy non-conservation, as the theoretical explanation shows.

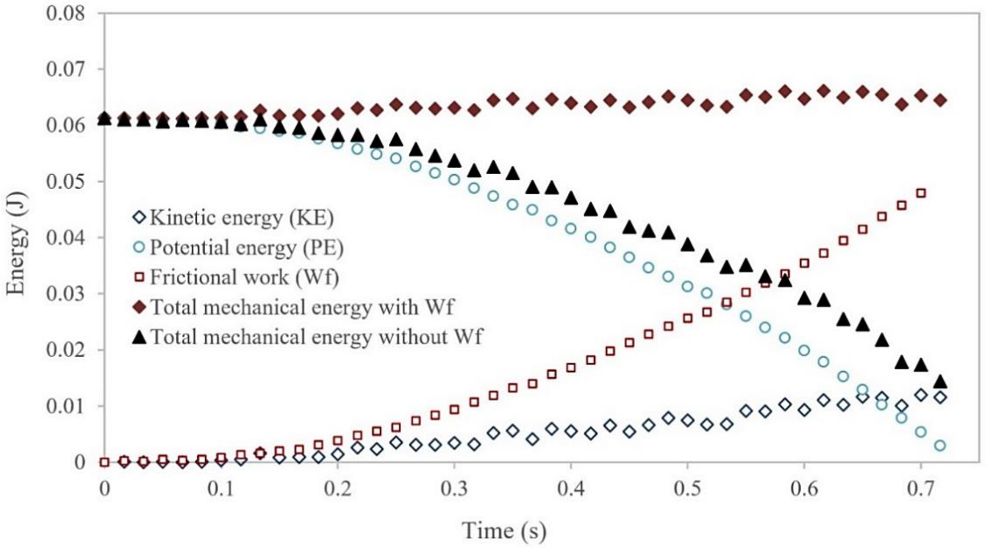


Figure 7 Energies comparison of a wooden block on inclined plane at a released angle of 30°.

CONCLUSIONS

In this study shows the results of using a high-speed camera and video analysis software to investigate the object moving on an inclined plane as the real world phenomenon, which can be clearly explained by the physics principle. The results indicate the coefficient of static and kinetic friction between the wooden plane and the object value of 0.48 and 0.40, respectively. The wooden plate acceleration increases in direct proportion to the releasing inclined plane angle. Total energy lost causes by the frictional force, indicating a non-conservation of energy. This high speed video technique can be used to increase the understanding of students and teachers, either about this phenomenon or others, which can be set in any classroom.

ACKNOWLEDGEMENTS

The authors would like to thanks Department of Physics and General Science, Faculty of Science and Technology, Chiang Mai Rajabhat University for the supporting facilities. Thanks are also given to Miss Sisara Chomphutong for useful fundamental data and Miss Cristina Madalina Dan for editing this manuscript.

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